Closing today: 4.3 (max/min)
Closing Fri: 4.4 (L'Hopital's rule)
Closing Mon: 4.4-5 (graphing)
Closing next Wed: 4.7(applied max/min)
Final Exam, Saturday, March 11
1:30-4:20pm, Kane 130

4.4 L'Hopital's Rule
First, recall as we discussed many times at the beginning of the term:
(Assuming f and g cont. at a)

$$\lim_{x \to a} \frac{f(x)}{g(x)} = ??$$

If $g(a) \neq 0$, then done! Ans = $\frac{f(a)}{g(a)}$.

If $g(a) = 0$ and $f(a) \neq 0$,
then examine each side of *x=a* (look at the sign) Ans = $\infty, -\infty$, or DNE.

If $g(a) = 0$ and $f(a) = 0$
then use algebra to rewrite in a form that 'cancels' the denominator.

L'Hopital's Rule (0/0 case)

Suppose $g(a) = 0$ and $f(a) = 0$ and *f* and *g* are differentiable at *x* = *a*, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Examples:

1. $\lim_{x \to 4} \frac{16 - x^2}{4 - x}$

2. $\lim_{x \to 0} \frac{\sin(x)}{x}$

3. $\lim_{x \to 0} \frac{\sqrt{1 + x} - 1}{x}$

Aside: Sketch of derivation

Assume $g(a) = 0$ and $f(a) = 0$

(These explanations are for the case when g'(a) is not zero).

Explanation 1 (def'n of derivative)

$$\frac{f'(a)}{g'(a)} = \frac{\lim_{x\to a} \frac{f(x) - f(a)}{x - a}}{\lim_{x\to a} \frac{g(x) - g(a)}{x - a}}$$

provided these limits exist we have:

$$\frac{f'(a)}{g'(a)} = \lim_{x\to a} \frac{\frac{f(x) - f(a)}{x - a}}{\frac{g(x) - g(a)}{x - a}}$$

$$= \lim_{x\to a} \frac{f(x) - f(a)}{g(x) - g(a)} = \lim_{x\to a} \frac{f(x)}{g(x)}$$

Explanation 2 (tangent line approx.):

The tangent lines for $f(x)$ and $g(x)$ at $x = a$ are

$$y = f'(a)(x - a) + 0$$
$$y = g'(a)(x - a) + 0$$

And we know these approximate the functions f(x) and g(x) better and better the closer x gets to a, so

Thus,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(a)(x - a)}{g'(a)(x - a)} = \frac{f'(a)}{g'(a)}$$

Sometimes you have to use it more than once.

Example:

$$\lim_{x \to 1} \frac{x - \sin(x-1) - 1}{(x-1)^3}$$

L'Hopitals rule can also be used directly for the ∞/∞ case

Example:

1. $\lim_{x \to \infty} \frac{5\mathrm{x} + 7}{6 + 13x}$

2. $\lim_{x \to \infty} \frac{\ln(\mathrm{x})}{x}$

3. $\lim_{x \to \infty} xe^{-3x}$

4. $\lim_{x \to \infty} \frac{3\mathrm{x} + 1}{\sqrt{9 + 4x^2}}$

Other indeterminant forms:

0·∞: (rewrite as a fraction)

$$\lim_{x\to 0^+} x\ln(x)$$

$$\lim_{x\to 0^+} x\,\mathrm{e}^{1/\mathrm{x}}$$

∞ – ∞ : (combine into a fraction)

(example from our midterm)

$$\lim_{t\to\infty} \frac{2}{t(1+3t)^2} - \frac{2}{t}$$

$\mathbf{0^0}, \infty^{\mathbf{0}}, \mathbf{1}^{\infty}$ **:** (Use ln( ))

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x$$

$$\lim_{x\to 0^+} x^x$$

Aside (you don't need to know this): The formula for compound interest is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

P = starting balance, A = end balance
r = annual rate, n = number of times interest is compounded each year,
t = number of years

In some bank accounts interest is computed once a month, for some every day, for some every second. If you wanted interested to always be computed (continuously), then the new formula would be

$$A = \lim_{n\to\infty} P\left(1 + \frac{r}{n}\right)^{nt}$$

You can use the techniques just discussed to find this limit and determine it is

$$A = Pe^{rt}$$

This is the continuous compounding interest formula.